A Numerical Method for the Solution of Junction of Cylindrical Waveguides

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Abstract

A numerical method for the junction of two cylindrical waveguides is developed by constructing a positive definite function from the boundary conditions and minimizing it with respect to the modal amplitudes. The junction of two rectangular waveguides is treated with mode and current probe excitations. Many types of diaphragms and junctions that are not available in the literature are treated. Of special interest is the case of a current element across the gap in a thin metallic post.

Introduction

The solution of various waveguide discontinuities has been treated in references [1,2,3,4] applying essentially the same mode matching techniques. This technique, however, exhibits a phenomenon of relative convergence first described in [1]. Other shortcomings have also been noted. Reference [4] shows that for the application of the Galerkin's method the ratio of the number of modes should be properly chosen. In addition, if it is applied to the thin iris discontinuity, the coefficient matrix of the modal amplitudes becomes singular unless an aperture field is assumed. Furthermore, these references treat different types of discontinuities, such as boundary enlargement and reduction, separately--requiring special computer programs--in each case.

We propose here a different approach to the mode matching technique, whereby the boundary conditions at the junction are satisfied in the mean square sense. This method has an intuitive appeal, is conceptually simple, does not have any of the problems of the relative convergence or singular matrices of the previous methods and does not depend on a judicious choice of the number of modes. It leads to a set of linear equations for the modal amplitudes which is stable, since the diagonal elements of the coefficient matrix usually have the largest magnitudes. This approach leads to general programs capable of handling large classes of problems. This is a highly desirable feature from the user's point of view in spite of its unavoidable inefficiencies. Because this approach does not rely heavily on the orthogonality of the modes or on their nondegeneracy, one need not orthogonalize the modes when degeneracies appear. This problem would be particularly troublesome in circular cylindrical waveguides.

Theoretical Development

We develop the method by considering the junction of two cylindrical waveguides as shown in Fig. 1, assuming the second guide matched although this is not a requirement imposed by the technique. We express the fields by their truncated modal expansions, and construct a so-called error function ε by integrating the magnitude squared of the expressions for the boundary conditions at the junction

$$\varepsilon=\alpha\int_{ap}|\underline{u}_z \times (\underline{H}^2-\underline{H}^1)-\underline{J}|^2 ds+\int_{ap}|\underline{u}_z \times (E^2-E^1)+\underline{M}|^2 ds+\int_{c(1)}|E^1|^2 ds + \int_{c(2)}|E^2|^2 ds \qquad (1)$$

where superscripts 1 and 2 represent the appropriate fields in the first and second guide respectively; $\underline{J}$ and $\underline{M}$ are electric and magnetic surface currents over some portion of the cross section at the junction; ap denotes the area of the apertures; c(1) and c(2) denote the surface area of the conducting diaphragms at the junction toward the first and second guide respectively. α is a weighting factor to balance the contribution of the error due to the H field.

The minimum of ε, which is a function of the modal coefficients, corresponds to the best matching of the fields at the junctions in the least square sense. Since ε is quadratic in the modal amplitudes, its unique minimum point can be obtained by solving a set of linear equations. We may express the boundary conditions as a linear matrix equation

$$LV = f \qquad (2)$$

where V is the modal coefficients vector and f is the excitation vector; the elements of L and f are functions of the mode functions. It can be shown that the set of linear equations which specifies the minimum of ε is the solution of the linear equation

$$\langle L^*,L\rangle V = \langle L^*,f\rangle \qquad (3)$$

$\langle L^*,L\rangle$ denotes the scalar multiplication of matrix L by its conjugate transpose where each element is integrated wherever valid [6]. The coefficient matrix is hermitian with the ensuing savings on computer storage requirement. Faster numerical routines are also available for the inversion of such matrices.

A variety of waveguide discontinuities such as bends, cascade of waveguides, bifurcations, multiple guide junctions, dielectric slabs and posts, metallic posts, metallic posts excited by a current probe across the gap, etc. can be readily treated by this method.

Parallel Plate Waveguide

As a simple example to check the method we have treated the step down discontinuity in a parallel plate waveguide. The formulation of the problem and the related computer program may be found in [6]. The results behave as expected. The modal magnitudes behave asymptoically as $A_n \sim n^{-5/3}$ for large n according to the edge condition.

Rectangular Waveguides

The junction of two offset rectangular waveguides is treated in detail. It is necessary to assume general fields as the sum of TE and TM modes to, say, z, the axis of propagation. The behavior of the inductive and capacitive diaphragms discussed in [5], and other special cases can be explained by the proper arrangement of the elements in the linear equation for the modal coefficients in [3]. For example, for the TE_{10} excitation the inductive discontinuity (cylindrical along the narrow side) generates TE_{m0} modes and no TM modes, whereas the capacitive junction (cylindrical along the broad side) generates TE_{1n} and TM_{1n} modes.

A computer program is available for the junction of two dissimilar rectangular waveguides. It assumes that the sides of the apertures are parallel to the coordinate axes. However, apertures with arbitrary boundaries can be readily treated by performing the relevant double integrations numerically. The double integrations reduce to single ones. The excitation may be by incident TE or TM modes or a constant current sheet at the junction. Whenever the two guides are identical we may effectively reduce the number of unknown modal coefficients by half using the symmetry. A computer program is also written for this case. The equivalent susceptances obtained from rather extensive computer program runs compare favorably with the available literature.

The sample examples presented here treat cases which have not been handled before. Figure 2 plots $|E|$ along the height at x =.3 for two capacitive metallic strips across the cross section of a rectangular waveguide. $|E|$ oscillates over the aperture, as it does in the related case of the parallel plate capacitive junction. It peaks up over the aperture toward the edges of the metallic strips and tends to zero over the strips. The relevant data and the equivalent susceptance are included in the caption. For the inductive junction, the variation of $|E|$ over the aperture along the broad side at the midpoint of the height resembles half a sine function. The detailed results for a variety of cases such as inductive and capacitive metallic strips, change in height or width, inductive and capacitive junction of two rectangular waveguides, etc. may be found in [6]. Figure 3 plots $-H_x^2 + H_x^1$ along y =.2 for a waveguide excited by a constant y-directed current probe across the gap in a zero thickness metallic post. The discontinuity in H_x at the junction should be equal to the y-directed induced current.

Weighting Factor

It may be shown that the minimum of the error function ε is a monotone increasing function of α, and that increasing α decreases the contribution of the partial error due to the H field and increases that due to the E field, and vice versa. On the other hand, we expect to increase the weight on the error due to the E field (decrease α) whenever the portions of the junction with large values of $|E|$ are covered with metallic conductor or the total aperture area is small. These points then indicate that there is at least a range of optimum values of α with respect to the number of modes selected.

A criterion for the selection of best α may be the conservation of real power. The real power and the equivalent susceptance are obtained from the propagating modal amplitudes. Consequently, the best α as far as the equivalent susceptance is concerned is that for which the conservation of real power is best satisfied. The normalized susceptance of asymmetric inductive strip and the discrepancy in the conservation of real power (p_r) are plotted in Fig. 4. For larger number of modes, the susceptance is less sensitive to the variation of α as indicated by the smaller slope of its curve. p_r progressively decreases for higher number of modes. It has a broad minimum against α, and for the corresponding values of α the susceptance changes only slightly. We may thus obtain at least the range of appropriate values of α. The correct value of the normalized susceptance is 1.08 and occurs at about the minimum of p_r.

Conclusion

This method is suitable for implementation on digital computers. It can be readily extended to different kinds of discontinuities. The procedure is to write all the boundary conditions in a matrix equation and perform the operations denoted in (3). As was mentioned before, besides having its own merits this procedure does not suffer the complications associated with other mode matching techniques.

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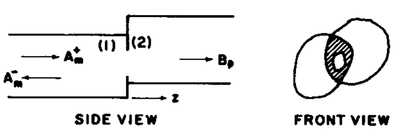

Fig. 1 Junction of two cylindrical waveguides.

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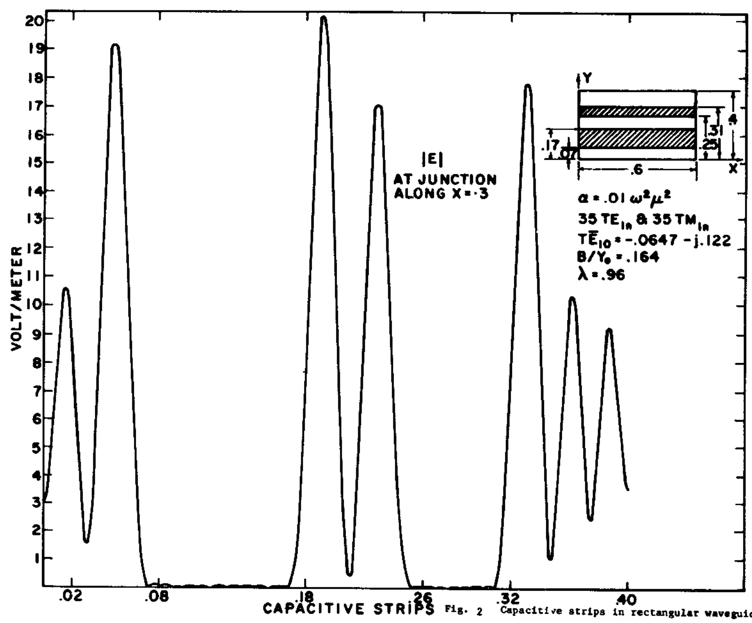

Fig. 2 Capacitive strips in rectangular waveguide.

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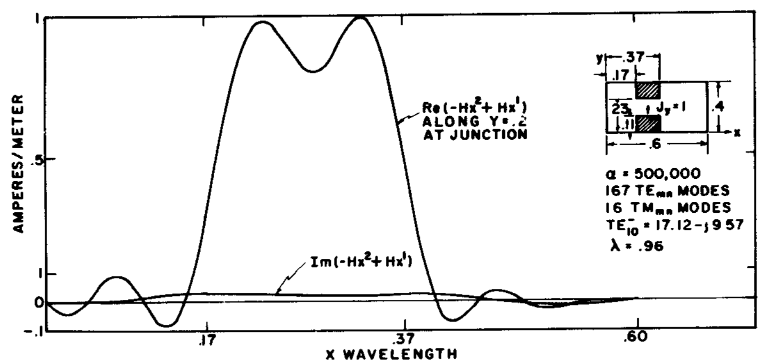

Fig. 3 Current probe excitation of a rectangular waveguide.

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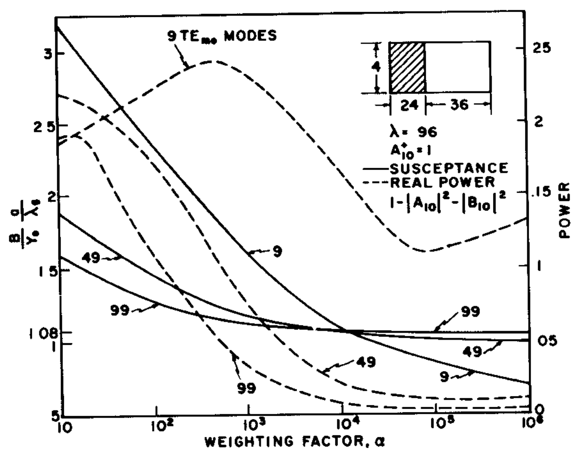

FIG 4 Susceptance and Real Power - Asymmetric Inductive Metallic Strip

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